Three Dimensional Large Amplitude Shallow Water Wave (Mechanisms and Mathematical Aspects of Nonlinear Wave Phenomena)

平川, 知明; 岡村, 誠

数理解析研究所講究録
第 1946 巻 2015 年 168-178

Three Dimensional Large Amplitude Shallow Water Wave

Tomoaki Hirakawa, Makoto Okamura
Interdisciplinary Graduate School of Engineering Science
Earth System Science Technology
Kyushu University

1. Introduction

Fluid phenomena around us often cannot be seen. Water waves are phenomena that commonly and clearly seen in our ordinary life. And such water wave motion is an attractive subject of fluid dynamics. Thorough understanding of the phenomenon is not simple and many works have been done on water wave.

Above all, three-dimensional interactions of solitary waves have been actively studied for the last several decades. Although Korteweg-de Vries (KdV) equation has been known for solitary waves, interactions of solitary waves attracted much attention after a proposition of Miles' theory (1981) and a derivation of approximated equations such as the Kadomtsev-Petviashvili (KP) equation (1970). An experiment of the reflection of solitary waves that corresponds to the interaction of an incident wave and its reflected wave was first conducted by Perroud (1956). A vertical reflecting wall was obliquely set in a water tank, so that an incident wave train generated along the tank can hit the reflecting wall at an angle and interact with its own reflected wave. He found out that when an incident wave impinges on the reflecting wall, the incident and the reflected waves strongly interact each other and the waveform of the interacting region resembles Mach reflection that was known as a phenomenon of compressible shock waves. Although the accuracy of the experiment was not high because of a deficient wave generator, measurement instruments and the small water tank, the realization of Mach reflection in the interaction of solitary waves was remarkable. After 20 years of the finding, Miles (1977) theoretically investigated reflections of solitary waves in the case of small amplitude shallow water waves $\epsilon \ll 1$ ($\epsilon = a_i/d$: $\epsilon =$ the nonlinear parameter, $a_i =$ the incident wave amplitude, $d =$ the uniform water depth). He obtained the solution for the oblique interaction between two solitons and provided an asymptotic description of the diffraction of a soliton at the corner of interaction angle $-\sqrt{3\epsilon} < \psi < \sqrt{3\epsilon}$. He predicted that the maximum run up of an incident wave becomes four times higher than the incident wave amplitude. Tanaka's numerical result for $\epsilon = 0.3$ shows better agreement with the prediction for non-grazing reflection than the prediction for strong resonant interaction. Stationary states were also not attained when it was close to $\psi = \sqrt{3\epsilon}$. Recently, Yeh, Li & Kodama (2010) have modified the interaction parameter used in Miles' theory. This modification affects some results when the interaction angle becomes relatively large. Using a new modified interaction parameter, the result proposed by Tanaka agrees with that of Miles' theory for non-grazing reflection better than that of the resonant interaction model. Li, Yeh & Kodama (2011) conducted experiments for reflection of an obliquely incident solitary wave at a vertical wall in the laboratory wave tank in the cases of $\epsilon = 0.076 - 0.367$ with $\psi = 30°$ and $20°$. This laboratory experiment presents results supporting Miles' theoretical predictions, as well as good agreement with Tanaka's numerical result.

In this study, we extend weakly nonlinear interactions of shallow water waves to strong nonlinear cases with a numerical scheme, such as the Newton method and the Galerkin method, and calculate periodic steady state solutions. A wave becomes a solitary wave in shallow water that is not periodic. Because such a non-periodic wave is difficult to be formed with periodic functions, we decide parameters so that a wave forms flat surface between adjacent wave crests. When the nonlinear parameter ϵ is small, most of our results well reproduce Miles' theory. However, in rather strong nonlinearity cases of $\epsilon > 0.1$, our results do not agree well with Miles' theory because the nonlinearity parameter ϵ in this study is out of Miles' approximation of weakly nonlinearity $\epsilon \ll 1$. This tendency is natural and already reported by other researchers.

2. Formulation of the problem

2.1 Fundamental equation for water waves

We consider free surface gravity waves on an inviscid, incompressible fluid of uniform depth and also irrotational flow is assumed. d is an uniform depth, ϕ is velocity potential, $z = \eta(x, y, t)$ is surface displacement, x, y are horizontal coordinates and z is vertical coordinate. Fundamental equations for water waves are written as follows

$$\Delta\phi = 0 \quad \text{in } z \leq \eta(x, y, t) \tag{2.1}$$

$$\phi_t+\frac{1}{2}\nabla\phi\cdot\nabla\phi+gz=0 \text{ on } z=\eta(x,y,t) \tag{2.2}$$

$$\frac{D}{Dt}\left(\frac{P}{\rho}\right)=\left(\frac{\partial}{\partial t}+\nabla\phi\cdot\nabla\right)\left[\phi_t+\frac{1}{2}\nabla\phi\cdot\nabla\phi+gz\right]=0 \text{ on } z=\eta(x,y,t) \tag{2.3}$$

$$\phi_z=0 \text{ as } z=-d \tag{2.4}$$

where g is the gravitational acceleration. Equation (2.3) is a Lagrange derivative of Bernoulli's equation (2.2). In stead of equation (2.3), equation of Lagrange derivative of a function $F(x,y,z,t)=z-\eta(x,y,t)=0$ on the free surface $z=\eta(x,y,t)$ can be used for the boundary condition:

$$\frac{DF}{Dt}=\left(\frac{\partial}{\partial t}+\nabla\phi\cdot\nabla\right)F=0 \text{ on } z=\eta(x,y,t) \tag{2.5}$$

However, this equation includes the free surface displacement $\eta(x,y,t)$ as an unknown, for this reason, we must reduce η by equation (2.2). In this point, because equation (2.3) is a form already η is reduced, (2.3) is useful.

2.2 Formulation for numerical calculation

We normalize variables as follows.

$$(x^*,y^*,z^*,H^*)=(Kx,Ky,Kz,KH),\qquad t^*=\omega t,\qquad \Phi^*=\frac{K^2}{\omega}\phi,\qquad G=\frac{K}{\omega^2}g, \tag{2.5}$$

where K is a wave number, ω is a frequency of an incident water wave. In order to calculate a steady progressive wave, we consider moving coordinate,

$$T=px^*-t^*,\qquad Y=qy^*,\qquad Z=z^*, \tag{2.6}$$

Here, $p=\sin\theta$, $q=\cos\theta$. When a wave number vector of solitary wave (k_x,k_y), we have relations:

$$k_x=K\sin\theta,\ k_y=K\cos\theta,\ \tan\theta=k_x/k_y,$$

Interactions pattern for solitary waves in T and Y-axis can be treated as period of 2π. Use equation (2.6) and substitute as

$$\Phi(Y,Z,T)=\Phi^*(x^*,y^*,z^*,H^*),\ H(Y,T)=H(x^*,y^*,t^*),$$

Using derived nondimensional variables, we change the form of the fundamental equation for water waves as follows.

$$p^2\Phi_{TT}+q^2\Phi_{YY}+\Phi_{ZZ}=0 \text{ for } Z\le H(Y,T), \tag{2.7}$$

$$P(Y,Z,T)=-\Phi_T+\frac{1}{2}\left(p^2{\Phi_T}^2+q^2{\Phi_Y}^2+{\Phi_Z}^2\right)+GZ=0 \text{ on } Z=H(Y,T), \tag{2.8}$$

$$\begin{aligned}Q(Y,Z,T)=\Phi_{TT}&+p^2\Phi_T(-2\Phi_{TT}+p^2\Phi_T\Phi_{TT}+q^2\Phi_Y\Phi_{YT}+\Phi_Z\Phi_{ZT})\\&+q^2\Phi_Y(-2\Phi_{YT}+p^2\Phi_T\Phi_{YT}+q^2\Phi_Y\Phi_{YY}+\Phi_Z\Phi_{YZ})\\&+\Phi_Z(-2\Phi_{ZT}+p^2\Phi_T\Phi_{ZT}+q^2\Phi_Y\Phi_{YZ}+\Phi_Z\Phi_{ZZ}+G)=0 \text{ on } Z\\&=H(Y,T),\end{aligned} \tag{2.9}$$

$$\Phi_Z=0 \text{ on } Z=-d, \tag{2.10}$$

We introduce the wave steepness

$$WS=\frac{1}{2}[H(0,0)-H(\pi,0)] \tag{2.11}$$

which is half of the difference between the peak $H(0,0)$ and the trough $H(\pi,0)$.
Assuming the velocity potential Φ as periodic

$$\Phi(Y,Z,T)=\sum_{k=0}^{N}\sum_{j=1}^{N}X(Z)\cos(kY)\sin(jT), \tag{2.12}$$

and we have a truncated series as follows:

$$\Phi(Y,Z,T)=\sum_{k=0}^{N}\sum_{j=1}^{N}A_{kj}[\cosh(\alpha_{kj}Z)+\sinh(\alpha_{kj}Z)\tanh(\alpha_{kj}d)]\cos(kY)\sin(jT), \quad \alpha_{kj}=\sqrt{p^2j^2+q^2k^2}. \tag{2.13}$$

3. Numerical scheme

At first, we numerically calculate the free surface displacement by applying Newton's method to the dynamic boundary condition (2.8). The recurrent formula for Newton's method to calculate the surface displacement H is

$$H_{n+1} = H_n - \frac{dZ}{dP(Y, H_n, T)} P(Y, H_n, T), \tag{3.1}$$

Next, we use Galerkin's method to obtain the independent relations for unknowns A_{kj}:

$$F_{lm}(A_{lm}, G) = \int_0^\pi dY \int_0^\pi dT Q(Y, H(A_{lm}, G), T) \cos(lY) \sin(mT) = 0, \tag{3.2}$$

(3.2) can be considered as *M*-point Fourier transform. Because when $l + m$ is odd, (3.2) is trivial, the number of independent relations (3.2) is $M(M + 1)/2$. Another independent relation is expressed as

$$W(A_{kj}, G) = 2WS - [H(0,0; A_{kj}, G) - H(\pi, 0; A_{kj}, G)] = 0, \tag{3.3}$$

which is a different expression of the wave steepness (2.11). Finally, we can obtain a sufficient number of independent relations and we can solve the nonlinear equations (3.2) and (3.3). Here, the third order approximation for short-crested wave calculated by Hsu *et. al.* (1979) is used as the initial solution of iteration. We stop the iteration if the difference between unknowns before and after iteration is smaller than 10^{-10}. The number of expansion terms of a solution $N(N + 1)/2$ is set as $N = 30$. The sampling point for Galerkin's method (two-dimensional Fourier transform) is set as 2^7.

4. Result

We calculated numerical solutions for various incident amplitudes, depths d and angles of incident waves θ by changing initial conditions such as wave steepness *WS*, an initial depths and angles of wave components θ_i. Note that wave components with an angle of θ_i not always form an incident wave with the same angle and we can evaluate the angle of an incident wave only from resulting waveforms. An incident wave amplitude is also evaluated from a result as;

$$a_i = H\left(\frac{\pi}{2}, \frac{\pi}{2}\right) - H\left(\frac{\pi}{2}, 0\right), \tag{4.1}$$

because of the symmetric condition. And we define the a_M as the center of the interacting region of wave profile as;

$$a_M = H(0,0) - H\left(\frac{\pi}{2}, 0\right), \tag{4.2}$$

that is the same as the definition of wave steepness. Therefore, we define the ratio α of the maximum wave amplitude to an incident wave amplitude as;

$$\alpha = \frac{a_M}{a_i} = \frac{H(0,0) - H\left(\frac{\pi}{2}, 0\right)}{H\left(\frac{\pi}{2}, \frac{\pi}{2}\right) - H\left(\frac{\pi}{2}, 0\right)} = \frac{WS}{H\left(\frac{\pi}{2}, \frac{\pi}{2}\right) - H\left(\frac{\pi}{2}, 0\right)}, \tag{4.3}$$

4.1 Weakly nonlinear cases

In this section, we discuss weakly nonlinear interactions. Asymptotic solutions of weakly nonlinear interactions are described by Miles' theory. 4.1, 4.2 and 4.3 show comparisons between Miles' theory and numerical results in cases of $d = 0.050$, $d = 0.070$ and $d = 0.090$ and the dependence of α respect to θ_i. Because of our symmetric assumption of a solution, a solution can not be found for $\kappa < 1$.

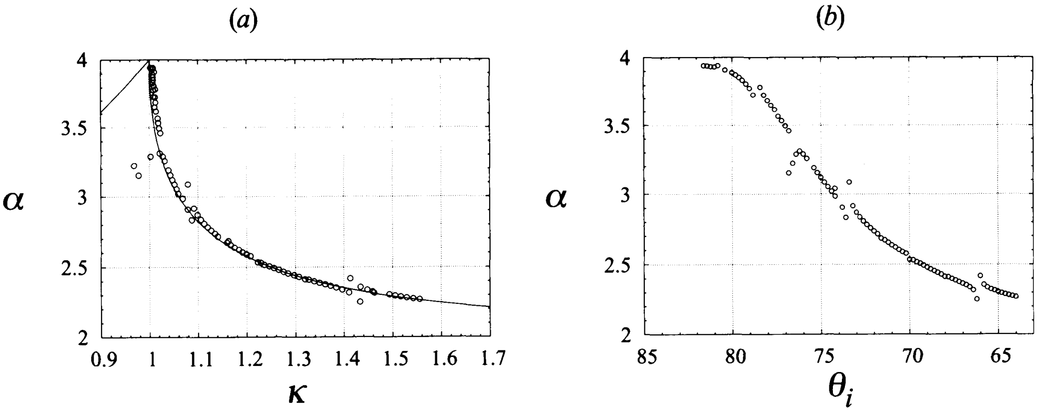
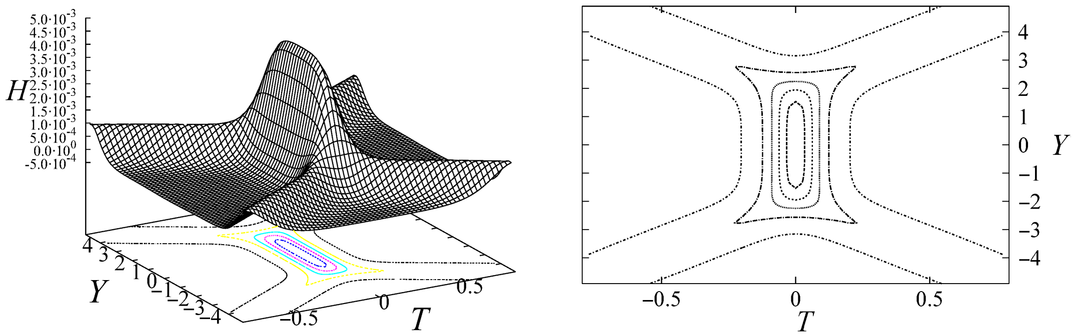


FIGURE 4.1. (*a*) The ratio $\alpha = a_M/a_i$ versus κ, (*b*) The ratio α versus the angle of wave components θ_i when $d = 0.050$

Most of our results are well reproduced by Miles' theory. However, some large and small discrepancies existed. We will study on this cause furthermore and compare it with harmonic resonance conditions Figure 4.2 shows wave a profile and a contour for $\kappa = 1.0$ and depths $d = 0.050$.



$\epsilon = 0.0256,\ \theta = 74.99$

FIGURE 4.2. Wave profiles and contours for $\kappa = 1.0$ and depths (a) $d = 0.050$

When $\kappa = 1.0$, interactions of two solitary waves are strong and an interaction region extend in Y direction. An interaction of two solitary waves is the same phenomena as reflection of an incident solitary wave so that Figure 4.2 implies a stem extends in perpendicular to the reflecting wall.

4.2 Harmonic resonance

In many conditions, unexpected rough waves and bad convergence appear. They seem sporadic and unavoidable problems. Most of interacting wave profiles that do not agree with that of Miles' theory, are contaminated by smaller wave components. Those contaminations occur sporadically and make it difficult for solutions to converge smoothly.

In order to investigated those deviations we sought regions enclosed with black frames (a) and (b) in Figure 4.3 which is in the case of fixed water depth $d = 0.050$ and $0.025 < \epsilon < 0.050$. Figure 4.4 shows enlarged figures of black frames (a) and (b) in Figure 4.3.

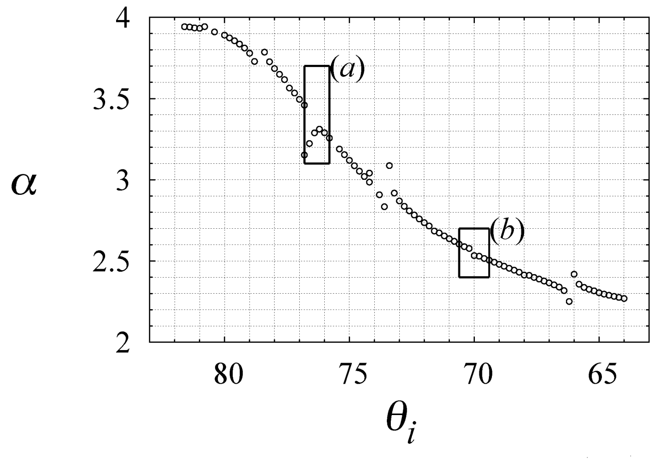


FIGURE 4.3. The ratio $\alpha = a_M/a_i$ versus frame angle θ_i (in degrees), when $d = 0.050$,.

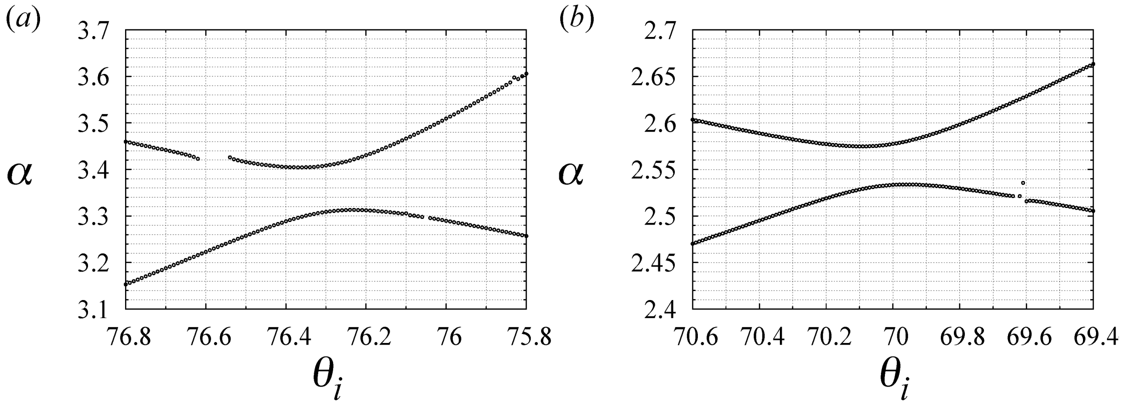
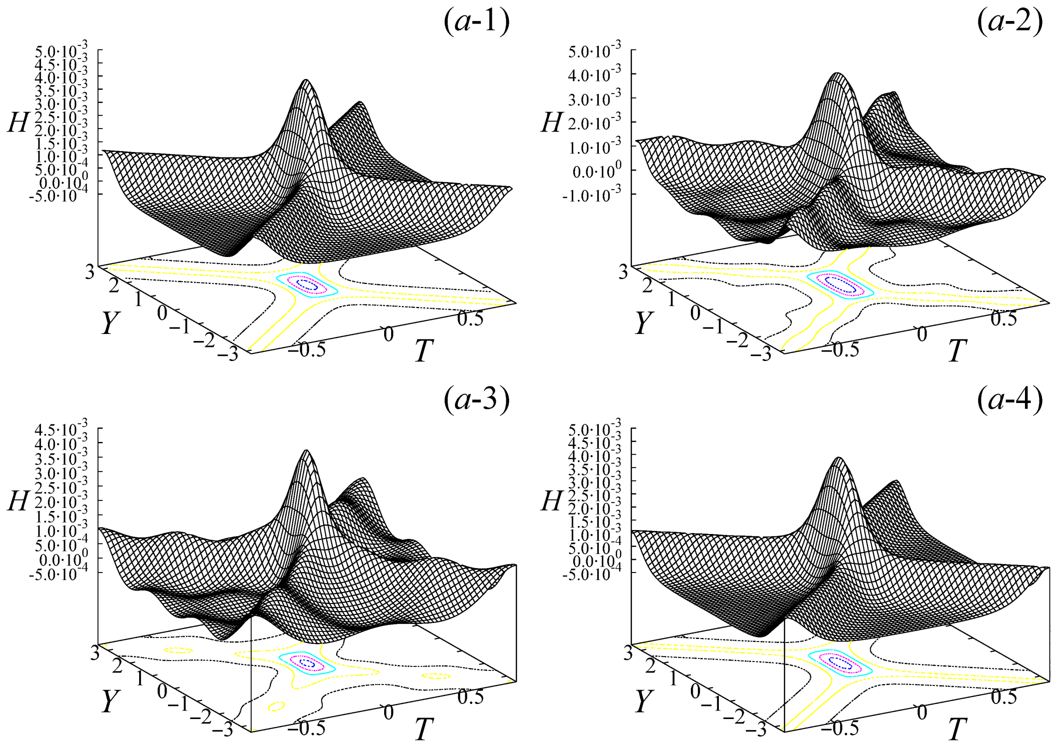


FIGURE 4.4. The ratio $\alpha = a_M/a_i$ versus the wave components angle θ_i (in degrees), when $d = 0.050$. (*a*) and (*b*) are enlarged figures of frames (*a*), (*b*) in FIGURE 4.3

Figure 4.4 shows we obtained solutions in pairs in most of wave components angles θ_i. At first, we checked difference between two wave profiles for several fixed wave components angle θ_i.



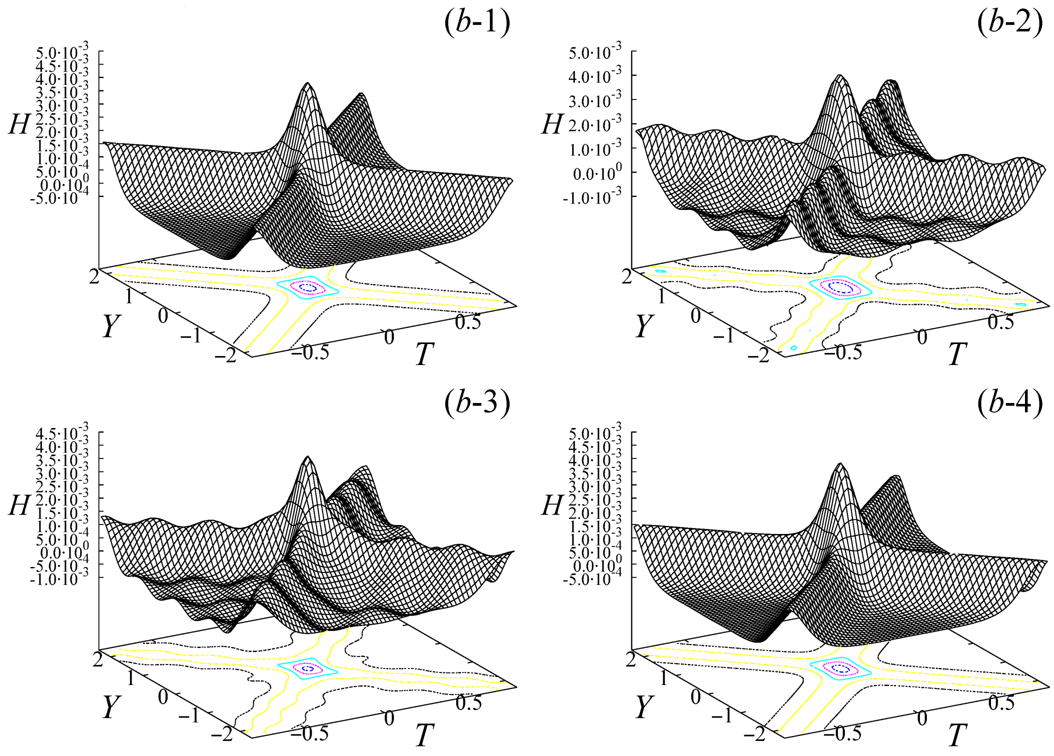
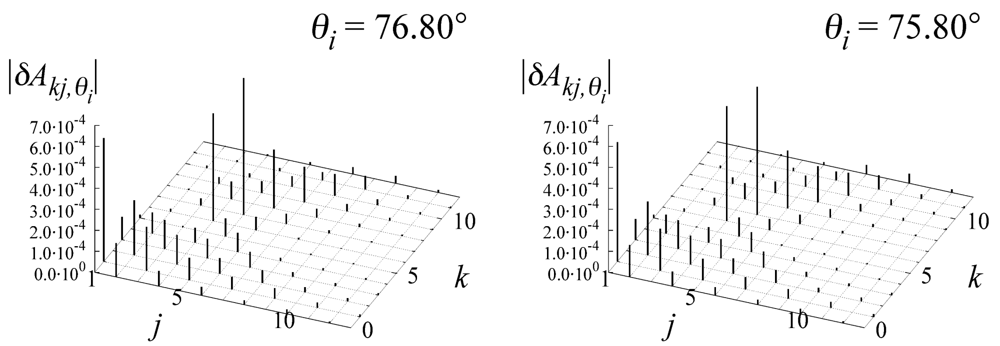


FIGURE 4.5. The wave profile of (*a*); $d = 0.050$, $\theta_i = 76.8$, (*b*); $d = 0.05$, $\theta_i = 75.8$, (*c*); $d = 0.09$, $\theta_i = 71.6$, (*d*); $d = 0.09$, $\theta_i = 70.8$.

In the next place, we check absolute values of differences between each modes $|\delta A_{kj,\theta_i}|$ of those pairs; (*a*-1)-(*a*-2), (*a*-3)-(*a*-4), (*b*-1)- (*b*-2) and (*b*-3)-(*b*-4) in Figure 4.5. $\delta A_{kj,\theta_i}$ is defined as;

$$\delta A_{kj,\theta_i} = A_{kj,\theta_i} - A_{kj,\theta_i} \tag{4.5}$$



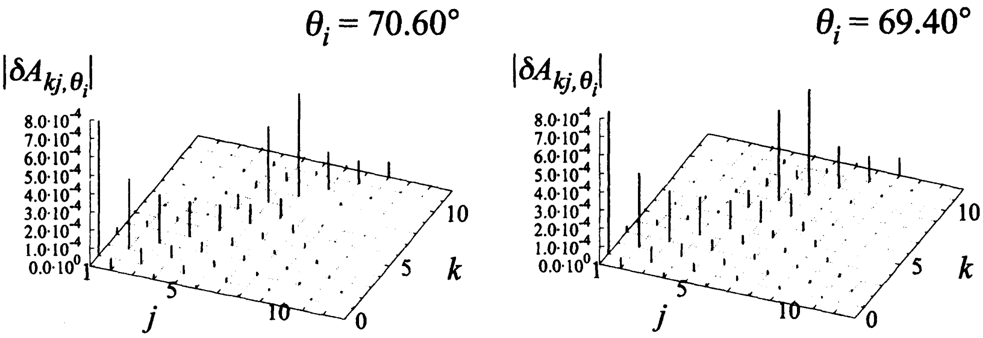


FIGURE 4.6. The absolute value of a difference coefficient $|A_{kj,\theta_i}|$ from $(k,j) = (0,1)$ to $(k,j) = (12,13)$

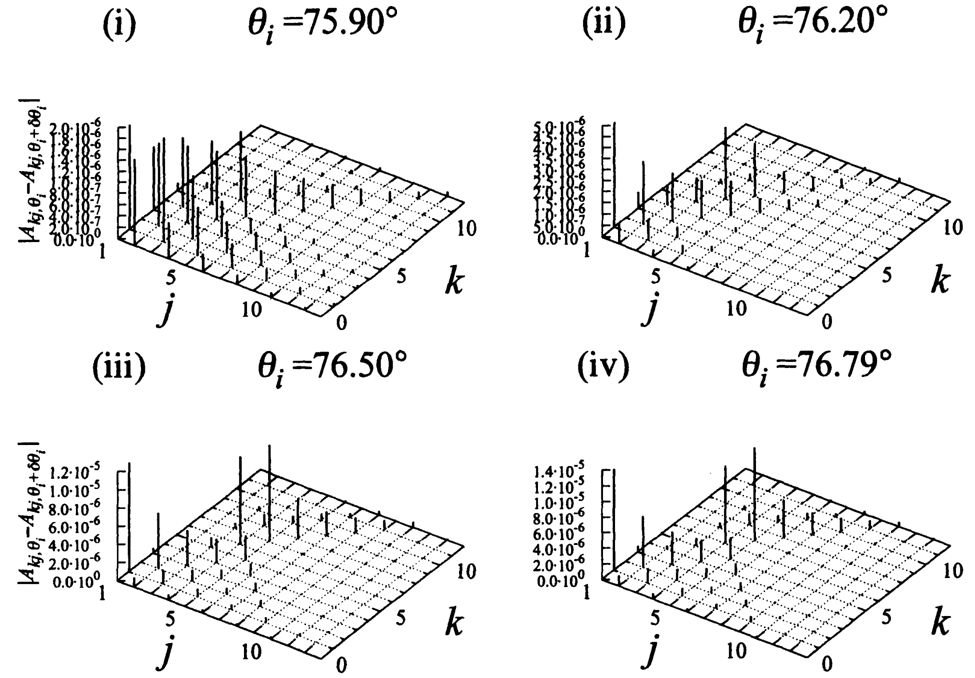


FIGURE 4.7. The absolute value of a difference coefficient $|A_{kj,\theta_i}|$ from $(k,j) = (0,1)$ to $(k,j) = (12,13)$: (i) $\theta_i = 75.90°$, (ii) $\theta_i = 76.20°$, (iii) $\theta_i = 76.50°$, (iv) $\theta_i = 76.79°$.

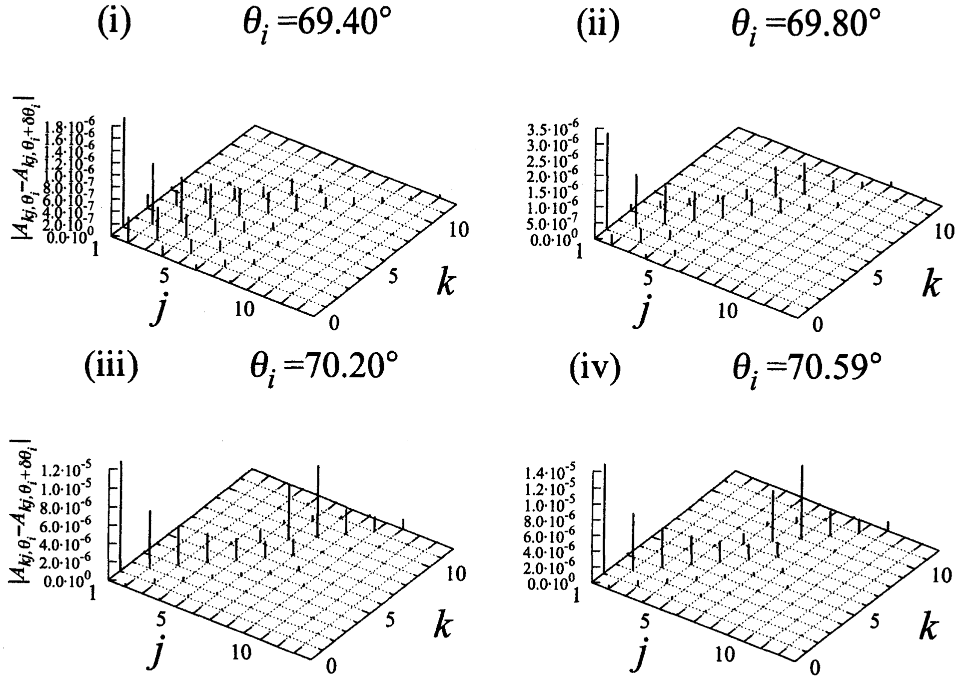


FIGURE 4.8. The absolute value of a diffenence coefficient $|A_{kj,\theta_i}|$ from $(k,j) = (0,1)$ to $(k,j) = (12,13)$: (i) $\theta_i = 69.40°$, (ii) $\theta_i = 69.80°$, (iii) $\theta_i = 70.20°$, (iv) $\theta_i = 70.59°$.

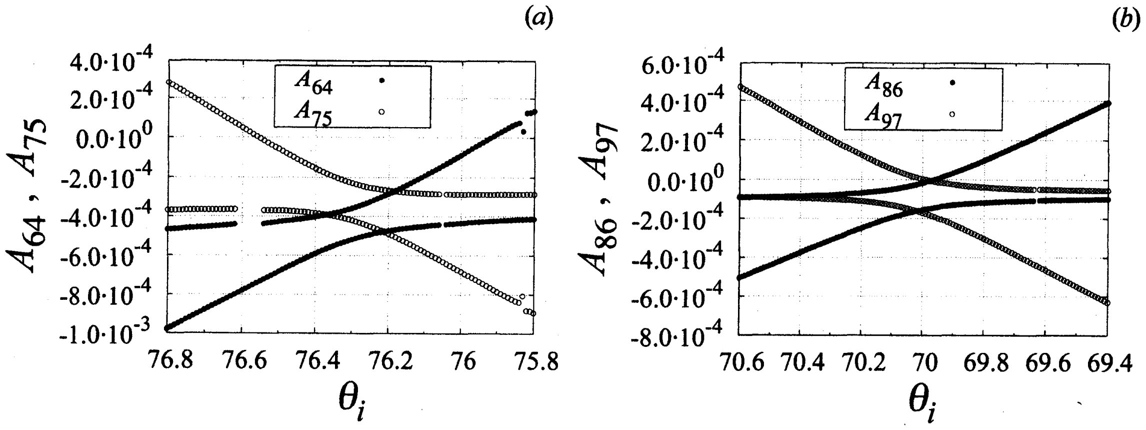


FIGURE 4.9. The value of a diffenence coefficient A_{kj,θ_i}. (a), (b) coresspond to enlarged figure of frames (a), (b) in FIGURE 4.3.

We make it clear weather those rough waves instabilities are occurred by an effect of a harmonic resonance or not. The harmonic resonance of three-dimensional interactions of water waves for finite depth and weakly nonlinearity cases is known to exist (Ioualalen 1996) if interactions of waves satisfy

$$\alpha_{kj}\tanh(\alpha_{kj}d) = j^2\tanh(d) \tag{4.6}$$

where solutions form are written as

$$\Phi(Y,Z,T) = \sum_{k=0}^{N}\sum_{j=1}^{N} A_{kj}[\cosh(\alpha_{kj}Z) + \sinh(\alpha_{kj}Z)\tanh(\alpha_{kj}d)]\cos(kY)\sin(jT), \quad \alpha_{kj} = \sqrt{p^2j^2 + q^2k^2}$$

Because equation (4.6) is only dependent on d, k, j and θ, when water depth d is given, we can make a table that shows harmonic resonance angle θ_{HR}. TABLE 4.1 shows harmonic resonance angles θ_{HR} when $d = 0.050$

k	j=1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	90.0	-	-	-	-	-	-	-	-	-	-	-	-	-
4	-	88.3	-	-	-	-	-	-	-	-	-	-	-	-
5	90.0	-	86.5	-	-	-	-	-	-	-	-	-	-	-
6	-	89.0	-	**84.2**	-	-	-	-	-	-	-	-	-	-
7	90.0	-	87.8	-	**81.6**	-	-	-	-	-	-	-	-	-
8	-	89.3	-	86.3	-	**78.6**	-	-	-	-	-	-	-	-
9	90.0	-	88.3	-	84.5	-	**75.3**	-	-	-	-	-	-	-
10	-	89.4	-	87.2	-	82.5	-	71.7	-	-	-	-	-	-
11	90.0	-	88.7	-	85.8	-	80.3	-	67.6	-	-	-	-	-
12	-	89.5	-	87.7	-	84.2	-	77.8	-	63.2	-	-	-	-
13	90.0	-	88.9	-	86.6	-	82.5	-	75.1	-	58.2	-	-	-
14	-	89.6	-	88.1	-	85.3	-	80.5	-	72.2	-	52.5	-	-
15	90.0	-	89.0	-	87.1	-	83.8	-	78.4	-	69.0	-	46.0	-
16	-	89.6	-	88.3	-	86.0	-	82.2	-	76.1	-	65.5	-	38.2

TABLE 4.1. Harmonic resonance angles θ_{HR} (in degrees) when $d = 0.050$. Harmonic resonance angles θ_{HR} when $(k,j) = (6,4), (7,5), (8,6)$ and $(9,7)$ are written in bold

We couldn't find good agreement between harmonic resonance angles θ_{HR} and the angle where those discrepancies occur. Harmonic resonance angles θ_{HR} when $(k,j) = (6,4), (7,5), (8,6)$ and $(9,7)$ are 84.2, 81.6, 78.6 and 75.3. Accordingly, angles of frames (*a*) and (*b*) in Figure 4.3 were no agreement with harmonic resonance angles. Because harmonic resonance angles are based on linear theory, nonlinearity might have effected on those angles.

4.3 Strong nonlinear cases

In this section, we discuss rather strong nonlinear interactions. In the case of rather large ϵ, results are unstable and solutions become more difficult to converge than weakly nonlinear cases. Bad convergences sporadically occur with changing conditions and often result in rough wave interaction profiles. Following figures show numerical results corresponding to $\epsilon = 0.2, 0.3$.

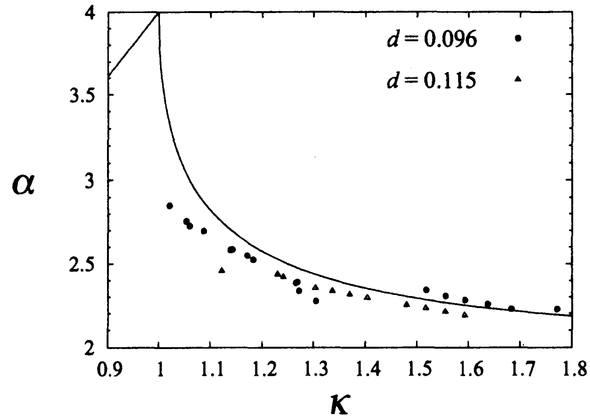


FIGURE 4.10. The ratio $\alpha = a_M/a_i$ versus the interaction parameter κ when $\epsilon = 0.20$.

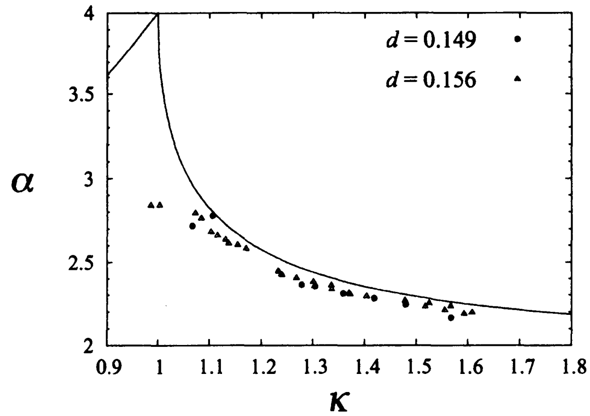
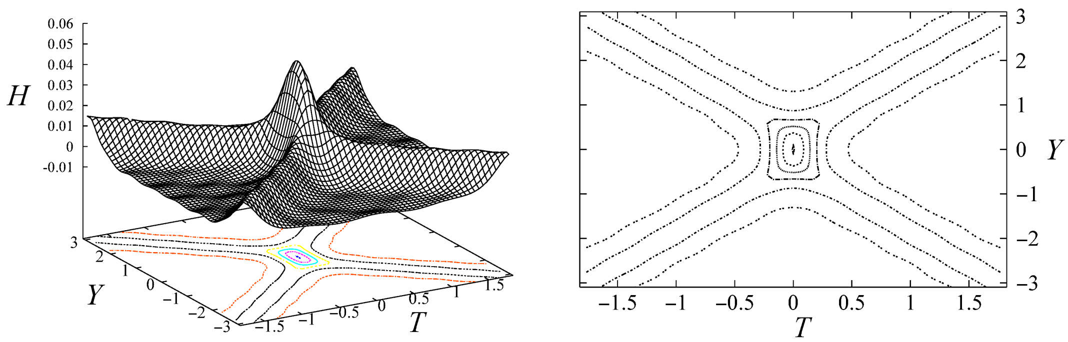


FIGURE 4.11. The ratio $\alpha = a_M/a_i$ versus the interaction parameter κ when $\epsilon = 0.30$.

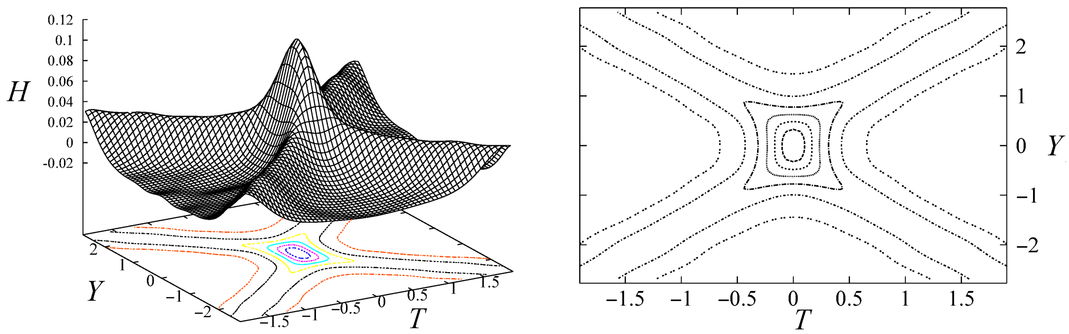
In the case of rather large ϵ, numerical results do not agree with Miles' theory because nonlinearity parameter ϵ is out of Miles' approximation of weakly nonlinearity $\epsilon \ll 1$.

Figure 4.10 and 4.11 differs depending on water depths d. However, even if the depth d is different, results should show the same result when ϵ are the same because the ratio $d/2\pi$ of the depth d to the wavelength 2π, which associated with solitary of waves, is set sufficiently small to be assumed the wavelength as infinity for waves in all cases, such a small change in depths d should not affect to the solutions.

For various conditions excluding the critical cases around $\kappa = 1$ and Mach reflection region $\kappa > 1$, there are cases that results shows rough waves or become unstable and do not converge enough which prevented us to obtain favorable data in succession. Those deviations are larger than those occur in weakly nonlinear cases.



(*a*)



(*b*)

FIGURE 4.12. Wave profiles and contours for $\kappa = 1.0$ and depths (*a*) $\epsilon = 0.2$ and (*b*) $\epsilon = 0.3$.

5. Conclusion

The scheme used in this research was successful to obtain solutions for three-dimensional large amplitude shallow water wave of ϵ up to 0.5. When ϵ is small, most of our numerical results show good agreement with Miles' theory. However, some discrepancies exist and we observed wave profiles became rough and difficult to convergence in those cases. They occur sporadically and we couldn't avoid such rough waves of solutions by reducing steps in calculations of the Newton method. And as ϵ increases, α starts decreasing and differences between the numerical results and Miles' theory increase and resulting wave profiles tend to be contaminated when κ is close to 1.

Existence of harmonic resonances has been known for periodic solutions of weakly nonlinear shallow water waves. Because the solutions used in this study are periodic, the solutions also have possibility to show harmonic resonances. Angles of frames (*a*) and (*b*) in Figure 4.3 were no agreement with harmonic resonance angles. Because harmonic resonance angles are based on linear theory, nonlinearity might have effected on those angles.

Reference

[1] Ablowitz, M.J. & Baldwin, D.E., 2012. Nonlinear shallow ocean wave soliton interactions on flat beaches.

[2] Concus, P., 1962. Standing capillary-gravity waves of finite amplitude. *Journal of Fluid Mechanics*, pp.568–576.

[3] Fultz, D., 1962. An experimental note on finite-amplitude standing gravity waves. *Journal of fluid mechanics*. Funakoshi, M., 1980. Reflection of Obliquely Incident Solitary Waves.pdf.

[4] Hammack, J., 1989. Two-dimensional periodic waves in shallow water. *Journal of Fluid Mechanics*, Hammack, J., 1995. Two-dimensional periodic waves in shallow water. Part 2. Asymmetric waves. *J. Fluid Mech*, 285.

[5] Hsu, J., 1979. Third-order approximation to short-crested waves. *Journal of Fluid Mechanics*

[6] Ioualalen, M. & Kharif, C., 1993. Stability of three-dimensional progressive gravity waves on deep water to superharmonic disturbances. *Eur. J. Mech. B. Fluids*, pp. 401–414.

[7] Ioualalen, M., Roberts, a. J. & Kharif, C., 1996. On the observability of finite-depth short-crested water waves. *Journal of Fluid Mechanics*, 322, pp.1–19.

[8] Ioualalen, M. & Kharif, C., 2006. On the subharmonic instabilities of steady three-dimensional deep water waves. *Journal of Fluid Mechanics*, 262(-1), p.265.

[9] Kadomtsev, B.B. & Petviashvili, V.I., 1970. On the stability of solitary waves in weakly dispersive media. *Sov. Phys. Doklady*, 15, pp.539–541.

[10] Knickerbocker Newell, 1985. Reflections from solitary waves in channels of dec.

[11] Kodama, Y., Oikawa, M. & Tsuji, H., 2009. Soliton solutions of the KP equation with V-shape initial waves. *Journal of Physics A: Mathematical and Theoretical*, 42(31), p.312001.

[12] Laitone, E. V, 1962. Limiting Conditionsfor CnoidalandStokesWaves. *Journal of Geophysical Research*, 67(4), pp.1555–1564.

[13] Li, W., Yeh, H. & Kodama, Y., 2011. On the Mach reflection of a solitary wave: revisited. *Journal of Fluid Mechanics*, 672, pp.326–357.

[14] Miles, J.W., 1977. Note on a solitary wave. *Journal of Fluid Mechanics*, pp.149–152.

[15] Miles, J.W., 1977a. Obliquely interacting solitary waves. *Journal of Fluid Mechanics*, 79.

[16] Miles, J.W., 1977b. On Hamilton's principle for surface waves. *Journal of Fluid Mechanics*, 83(01), p.153.

[17] Miles, J.W., 1977c. Resonantly interacting solitary waves. *Journal of Fluid Mechanics*, 79, pp.171–179.

[18] Miles, J.W., 1981. The Korteweg-de Vries equation: a historical essay. *Journal of Fluid Mechanics*, 106(-1), p.131.

[19] Okamura, M., 2010. Almost limiting short-crested gravity waves in deep water. *Journal of Fluid Mechanics*, 646, p.481.

[20] Tanaka, M., 1993. Mach reflection of a large-amplitude solitary wave. *Journal of Fluid Mechanics*, 248, pp.637–661.

[21] Tanaka, M., 1986. The stability of solitary waves. *Phys. Fluids*.

[22] Tsai, C.-P. & Jeng, D.-S., 1994. Numerical Fourier solutions of standing waves in finite water depth.

[23] Wineberg, S. & McGrath, J., 1991. Implicit spectral methods for wave propagation problems. *Journal of Fluid Mechanics*, 336.

[24] Yeh, H., Li, W. & Kodama, Y., 2010. Mach reflection and KP solitons in shallow water., p.16.

Interdisciplinary Graduate School of Engineering Science
Kyushu University
6-1 Kasuga-Koen, Kasuga 816-8580
Japan
E-mail address: t-hirakawa@riam.kyushu-u.ac.jp

九州大学・総合理工学府大気海洋環境システム学専攻　平川　知明，岡村　誠